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MATHEMATICS TARGET THE EXCELLENCE KEY AGYAT GUPTA (M.Sc., M.Phil.) **CODE:- AG-TS-4-9999** 

# **REGNO:-TMC -D/79/89/36**

### **GENERAL INSTRUCTIONS:**

- 1. All question are compulsory.
- 2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section - B comprises of 12 questions of 4 marks each and Section - C comprises of 7 questions of 6 marks each.
- 3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
- There is no overall choice. However, internal choice has been provided in 4. 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
- Use of calculator is not permitted. 5.
- Please check that this question paper contains 5 printed pages. 6.
- 7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

## Pre-Board Examination 2012 -13

Time : 3	Time : 3 Hours अधिकतम समय		
Maxim	Maximum Marks : 100 अधिकतम अंक : 10		
Total N	o. Of Pages :6		कुल पृष्ठों की संख्या : 6
CLAS	SS – XII	CBSE	<b>MATHEMATICS</b>
NOTE : - THE QUESTION PAPER WILL INCLUDE QUESTION(S)			
BASED ON VALUES TO THE EXTENT OF 5 MARKS.			
PART – A			
Q.1	Let $A = [a_{ij}]$ be a s	square matrix of orde	er $3 \times 3$ , and $C_{ij}$ denote cofactor

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	of $a_{ij}$ in A. If $ A =5$ , write the value of			
	$a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$ . Answer-:5			
Q.2	If A is square matrix of order 3 such that $ AdjA  = 225$ . Find $ A^T $ .			
Q.3	The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point (a) (1,2) (b) (2,1) (c) (1,-2) (d) (-1,2) ans : a			
Q.4	Construct a $2 \times 2$ matrix whose elements <i>aij</i> are given by			
	$a_{ij} = \begin{cases} \frac{ -3i+j }{2} & ifi \neq j \\ (i+j)^2 & ifi = j \end{cases} \cdot \frac{4 + 1/2}{5/2 + 16}$			
Q.5	Find the value of derivative of $\tan^{-1}(e^x)$ w.r.t. x at the point x = 0. ans : $1/2$			
NOTE	Fill in the blanks in Questions 6 to 8.			
Q.6	$\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx = \dots \qquad \text{ans : } x + c$			
Q.7	If $a = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\hat{b} = 3\hat{i} - 2\hat{j} + \lambda\hat{k}$ are perpendicular each other, then $\lambda = \dots = 1$ ans : $\lambda = -2$			
Q.8	The projection of $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$ along $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is ans :1/7			
Q.9	The 2 vectors $\hat{j} + \hat{k}$ and $3\hat{i} - j + 4\hat{k}$ represents the two sides AB and AC, respectively of a $\triangle$ ABC. Find the length of the median through Aans : Median AD is given by			
Q.10	Evaluate $\int_{-5}^{5} (\sin^{-83} x + x^{123}) dx$ . ans :0			

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	PART – B	Q.16	Water is dripping		
Q.11	Solve $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$ . Ans. $x = 0, \frac{1}{2}$		at the vertex of the slant height of w		
	OR		slant height, whe		
	Solve the equation : $\sec^{-1} \frac{x}{a} - \sec^{-1} \frac{x}{b} = \sec^{-1} b - \sec^{-1} a$ .		Solution Given t		
	$x = \pm ab$		conical vessel.		
Q.12	Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$ where a,b,c are in A.P.		From the Fig.6.2, <i>l</i> =		
	x+3 $x+4$ $x+c$		1		
Q.13	Evaluate: $\int_{-\infty}^{1} x(\tan^{-1} x)^2 dx.$		Therefore, $v = \frac{1}{3}\pi$		
	Solution $I = \int_{0}^{1} x(\tan^{-1}x)^2 dx$ . $= \frac{\pi^2}{32} - I_1$ , where $I_1 = \int_{0}^{1} \frac{x^2}{1+x^2} \tan^{-1}x dx$		$\frac{dv}{dt} = \frac{\sqrt{3}\pi}{8}l^2$		
	Integrating by parts, we have $I = \frac{x^2}{2} \Big[ (\tan^{-1} x)^2 \Big]_0^1 - \frac{1}{2} \int_0^1 x^2 \cdot 2 \frac{\tan^{-1} x}{1 + x^2} dx$ Now $I_1 = \int_0^1 \frac{x^2 + 1 - 1}{1 + x^2} \tan^{-1} x  dx$		Therefore, $1 = \frac{\sqrt{3}\pi}{8}$		
	$= \frac{\pi^2}{32} - \int_0^1 \frac{x^2}{1+x^2} \cdot \tan^{-1} x  dx \qquad = \int_0^1 \tan^{-1} x  dx - \int_0^1 \frac{1}{1+x^2} \tan^{-1} x  dx$		$\Rightarrow \frac{d}{d}$		
Q.14	If $x = 2\cos\theta - \cos 2\theta$ & $y = 2\sin\theta - \sin 2\theta$ find $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{2}$ .		Therefore, the rate of		
	Ans: $\frac{d^2 y}{dx^2}$ at $\theta = \frac{\pi}{2}$ is $\frac{3}{8} \sec^3 \frac{3\pi}{4} \csc \frac{\pi}{4} = \frac{-3}{2}$		Find the interval		
Q.15	If $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$ then $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ .		$x^3 + \frac{1}{x^3}, x \neq 0$		

w. agyatgupta.com; Email:agyat99@gmail.com ng out at a steady rate of 1 cu cm/sec through a tiny hole the conical vessel, whose axis is vertical. When the water in the vessel is 4 cm, find the rate of decrease of here the semi vertical angle of the conical vessel is  $\frac{\pi}{6}$ . that  $\frac{dv}{dt} = 1 \text{ cm}^{3/\text{s}}$ , where v is the volume of water in the l = 4cm,  $h = l \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}l$  and  $r = l \sin \frac{\pi}{6} = \frac{l}{2}$ .  $\frac{1}{3}\pi r^2 h = \frac{l^2}{3} \frac{\sqrt{3}}{4} l \frac{\sqrt{3}}{2} l \frac{\sqrt{3}}{24} l^3.$  $\frac{l^2}{dt} \frac{dl}{dt}$  $\frac{3\pi}{3}$  16. $\frac{dl}{dt}$  $\frac{dl}{dt} = \frac{1}{2\sqrt{3\pi}}$  cm/s. Fig. 6.2 e of decrease of slant height =  $\frac{1}{2\sqrt{3\pi}}$  cm/s. OR als in which the function f given by f(x) =0 is (i) increasing (ii) decreasing . ans :

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	$f(x) = x^3 + \frac{1}{x^3}  \Rightarrow  f'(x) = 3x^3 - \frac{3}{x^4} \qquad =  \frac{3(x^6 - 1)}{x^4} = \frac{3(x^2 - 1)(x^4 + x^2 + 1)}{x^4}$			
	Thus $f$ is increasing in $(-\infty, -1) \cup (1, \infty)$			
	Thus $f(x)$ is decreasing in $(-1, 0) \cup (0, 1)$			
Q.17	Obtain a differential equation of the family of circles touching the x-			
	axis at origin. Ans: Equation of circle : $x^2 + (y-a)^2 = a^2$ Required			
	differential eqn $(x^2 - y^2)y_1 = 2xy$			
Q.18	Evaluate: $\int \frac{dx}{(\sin x + \sin 2x)} \cdot \frac{\sin 2x}{\sin 2x}$			
	$I = \int \frac{dx}{\sin x (1 + 2\cos x)} = \int \frac{\sin x  dx}{\sin^2 x (1 + 2\cos x)}$			
	$= \int \frac{\sin x  dx}{(1 - \cos x)(1 + \cos x)(1 + 2\cos x)}$			
	Now differential coefficient of $\cos x$ is $-\sin x$ which is given			
	in numerator and hence we make the substitution $\cos x = t \Rightarrow -\sin x  dx = dt$			
	$\therefore I = -\int \frac{dt}{(1-t)(1+t)(1+2t)}$			
	We split the integrand into partial fractions			
	$\therefore I = -\int \left[\frac{1}{6(1-t)} - \frac{1}{2(1+t)} + \frac{4}{3(1+2t)}\right] dt \text{ etc.}=$			
	$\frac{1}{6}\log(1-\cos x) + \frac{1}{2}\log(1+\cos x) - \frac{2}{3}\log(1+2\cos x).$			
	OR			

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	Evaluate: $\int \cos 2\theta \log \left( \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta$ . We know that		
	$\log\left(\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}\right) = \log\left(\frac{1 + \tan\theta}{1 - \tan\theta}\right) = \log\tan\left(\frac{\pi}{4} + \theta\right)$		
	$\int \sec\theta  d\theta = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$		
	$\therefore \int \sec 2\theta  d\theta = \frac{1}{2} \log \tan \left( \frac{\pi}{4} + \theta \right)$		
	$\therefore 2 \sec 2\theta = \frac{d}{d\theta} \log \tan \left(\frac{\pi}{4} + \theta\right) \qquad \dots \dots (i)$		
	Integrating the given expression by parts, we get		
	$I = \frac{1}{2}\sin 2\theta \log \tan\left(\frac{\pi}{4} + \theta\right) - \frac{1}{2}\int \sin 2\theta \cdot 2\sec 2\theta d\theta \qquad \text{by} \qquad (i)$		
	$= \frac{1}{2}\sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta\right) - \int \tan 2\theta  d\theta = \frac{1}{2}\sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta\right) - \frac{1}{2}\log \sec 2\theta.$		
Q.19	Find the particular solution of the differential equation: $(x-\sin y)dy + (\tan y)dx = 0$ : given that $y = 0$ when $x = 0$ . Which vehicle		
	you will suggest for long trip for 50 persons ? ans :		
	The given diff. eqn. can be written as		
	$\frac{dx}{dy}$ + (coty) x = cosy		
	$I.F = e^{\int \cot y  dy} = e^{\log \sin y} = \sin y$		
	$\therefore$ The solution is x siny = $\int siny \cos y  dy + C$		
	$=\frac{1}{2}\int \sin 2y  dy + c  \text{or } x  \sin y = \frac{-1}{4}\cos 2y + C$		

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	It is given that y=0, when x=0 $\therefore x \sin y = \frac{1}{4} (1 - \cos 2y) = \frac{1}{2} \sin^2 y$			
	$C-\frac{1}{4}=0 \Rightarrow C=1/4$ $\Rightarrow 2x = siny is the reqd. solution$			
	ANS for a trip of 50 persons a bus should be used. As it has several			
	advantage (a) it can carry many people at a time			
	(b) it is comfortable like Volvo etc .and safe There can be multiple			
	answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks			
	would be given for all sensible answers.			
Q.20	Let $f(x) = x x $ , for all $x \in R$ Discuss the derivability of $f(x)$ at $x = \int_{x}^{x} dx  dx$			
	Solution We may rewrite $f$ as $f(x) = \begin{cases} x^2, \text{ if } x \ge 0\\ -x^2, \text{ if } x < 0 \end{cases}$			
	Now $Lf'(0) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{-h^2 - 0}{h} = \lim_{h \to 0^-} -h = 0$			
	Now $\mathbb{R}f'(0) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{h^2 - 0}{h} = \lim_{h \to 0^-} h = 0$			
	Since the left hand derivative and right hand derivative both are equal, hence $f$ is differentiable at $x = 0$ .			
Q.21	Find the equation of the plane containing the two lines, $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} + 2\hat{k} + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$ .			
	Find the distance of this plane from origin and also from the point (1, 1, 1).ans :			
	$\vec{n} = 20i + 10 j - 8k \& \vec{r} \bullet (20i + 10 j - 8k) = 74 \Rightarrow \vec{r} \bullet (10i + 5 j - 4k) = 37$			
	Hence equation of required plane is $10 x + 5 y - 4 z = 37$ & Distance			
	from (1, 1, 1) to the plane is $\frac{37}{\sqrt{141}} \& \frac{26}{\sqrt{141}}$ .			
Q.22	In answering a question on a MCQ test with 4 choices per question, a			
	student knows the answer, guesses or copies the answer. Let <sup>1</sup> / <sub>2</sub> be the			

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probability that he knows the answer, <sup>1</sup>/<sub>4</sub> be the probability that he guesses and <sup>1</sup>/<sub>4</sub> that he copies it. Assuming that a student, who copies the answer, will be correct with the probability <sup>3</sup>/<sub>4</sub>, what is the probability that the student knows the answer, given that he answered it correctly? Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years? Ans: Let A be the event that he knows the answer, B be the event that he guesses and C be the event that he copies. Then,  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{4}$  and  $P(C) = \frac{1}{4}$  Let X be the event that he has answered correctly. Also, P(X/A) = 1,  $P(X/B) = \frac{1}{4}$  and  $P(X/C) = \frac{3}{4}$  Thus, Required probability =  $P(A/X) = \frac{1}{2} \times 1$ 

 $\frac{P(X/A) \times P(A)}{P(X/A) \times P(A) + P(X/B) \times P(B) + P(X/C) \times P(C)} =$ 

 $=\frac{2}{1, 1, 3}$ 

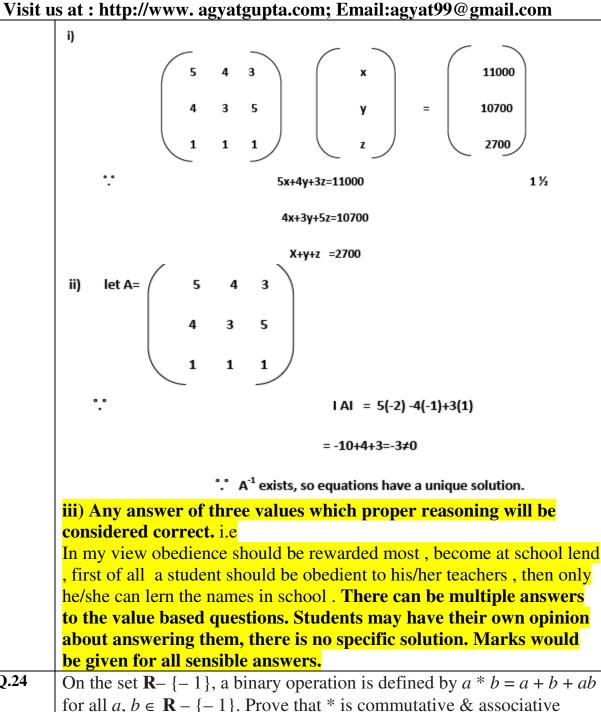
 $\frac{1}{2} + \frac{1}{16} + \frac{2}{16} = \frac{2}{3}$  ans : If Arjun copies the answer, he will be violating the value of honesty in his character. He should not guess the answer as well as that may fetch him negative marking for a wrong guess. He should accept the question the way it is and leave it. There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.

#### OR

An insurance company insured 2000 cyclists, 4000 scooter drivers and 6000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motorbike driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What

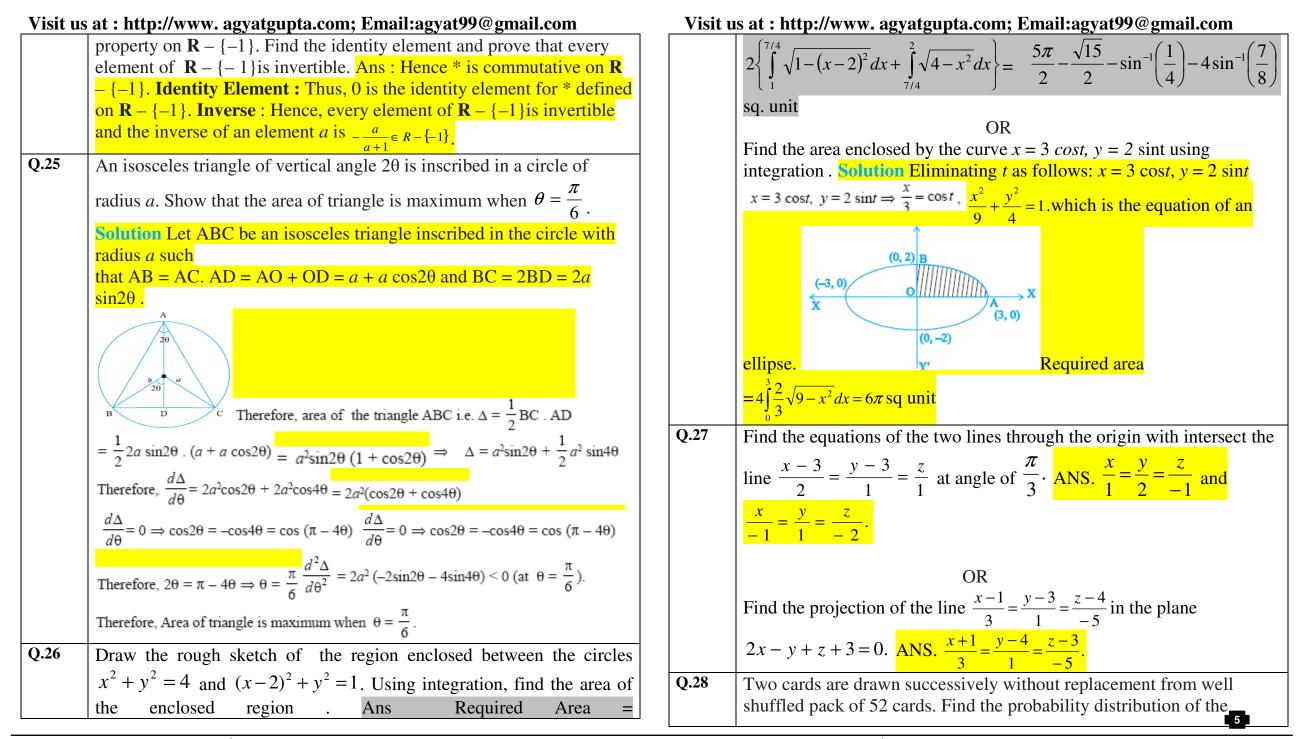
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/0 _ 0 _ 0		/= = = =	
	is the probability that he is a scooter driver? Which mode of transport		i)
	would you suggest to a student and why? Ans: Let the events defined		-
	are E1: Person chosen is a cyclist		
	E2: Person chosen is a scoter-driver		
	E3: Person chosen is a motorbike driver		
	A: Person meets with an Accident		
	$P(E1) = 1/6, P(E2) = 1/3, P(E3) = \frac{1}{2}, P(A/E1) = 0.01, P(A/E2) = 0.03,$		
	P(A/E3) = 0.152, P(E1/A) = P(A/E1).P(E1)/P(A) = 1/2		-
	Suggestion: Cycle should be promoted as it is good for		
	i. Health ii. No pollution iii. Saves energy( no petrol). There can be		
	multiple answers to the value based questions. Students may have		
	their own opinion about answering them, there is no specific		
	solution. Marks would be given for all sensible answers.		ii) l€
	PART – C		
Q.23	Two schools A and B decided to award prizes to their students for three		
	values honesty (x), punctuality (y) and obedience (z). School A decided		
	to award a total of Rs. 11000 for the three values to 5, 4 and 3 students		
	respectively while school B decided to award Rs. 10700 for the three		:
	values to 4, 3 and 5 students respectively. If all the three prizes together amount to Rs. 2700, then.		
	i. Represent the above situation by a matrix equation and form Linear		
	Equations using matrix multiplication.		
	ii. Is it possible to solve the system of equations so obtained using		<mark>iii) An</mark>
	Matrices ?		<mark>consid</mark>
	iii. Which value you prefer to be rewarded most and why? Ans:		In my
			<mark>, first c</mark>
			he/she
			to the
			about
			be give
		Q.24	On the

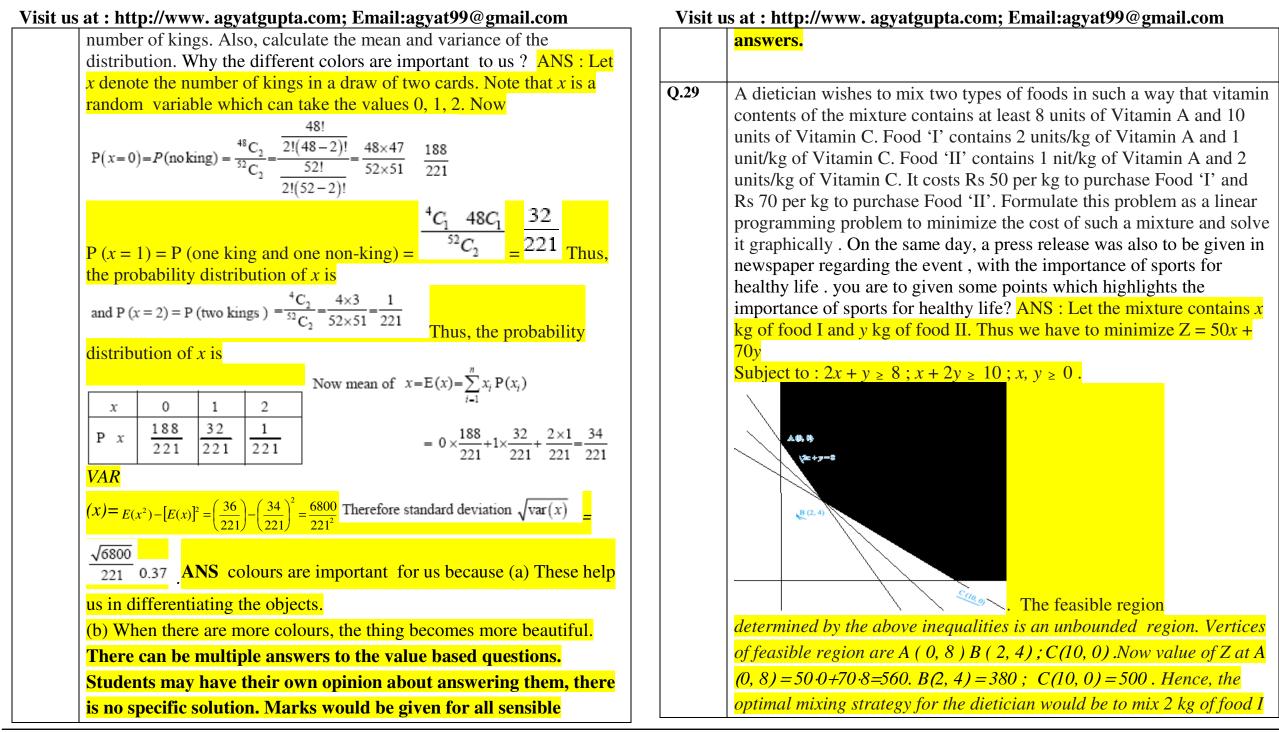


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and 4 kg of food II to get the minimum cost of the mixture i.e Rs 380. **ANS** sports are very important to lead a healthy life. These help to keep overselves (a) Fresh (b) healthy (c) smarter (d) intelligent (e) fit . **There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.** 

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